The Specials Market for U.S. Treasury Securities and the Federal Reserve’s Securities Lending Program

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Abstract

This paper examines the relationship between the Federal Reserve's securities lending program and the over-the-counter market for special collateral repurchase agreements. The characteristics of dealer borrowings from the Fed suggest that the two markets are fairly well integrated. The incidence and extent of borrowings, and the fees paid to borrow securities, are broadly consistent with expectations based on closely integrated markets. However, there is some evidence that transaction costs, the design of the Fed's program, and other market frictions sometimes lead dealers to borrow securities from the Fed when private sector borrowings are more attractively priced and sometimes deter dealers from exploiting arbitrage opportunities.

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Financial economists have paid increasing attention to markets for borrowing and lending securities over the past decade. In a path-breaking article, Duffie (1996) describes how fee income from lending a U.S. Treasury note supplements the security's principal and interest payments. He hypothesizes that expectations of high loan fees increase the equilibrium price of a security above that which would prevail in the absence of such fees. Jordan and Jordan (1997) confirm the hypothesis for Treasury notes and Krishnamurthy (2002) provides corroborating evidence for bonds. Keane (1996) observes that loan fees for recently auctioned Treasury notes follow a predictable pattern over the course of an auction cycle and Cherian, Jacquier, and Jarrow (2002) analyze the implications of the pattern for cash market prices.1

Market participants borrow Treasury securities to deliver against short sales and to cure settlement fails. A short sale is a sale of securities that the seller does not own and has to borrow to make delivery. It may be executed in the belief that the price of a security will be lower in the future, in the course of putting on a “spread” trade where one security is purchased and another sold short in anticipation of a change in the relative prices of the securities,2 as a hedge against another position (such as a long position in corporate bonds or mortgage-backed securities),3 or as a result of a dealer accommodating the purchase interests of a customer. The short seller returns the borrowed securities when it closes out its short position with an offsetting purchase.

A settlement fail occurs when a seller does not deliver securities on the date originally scheduled by the buyer and seller.4 Fails occur most commonly when a dealer cannot deliver

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2 Spread trades are discussed in Garbade (1996, ch. 11).


securities because it failed to receive the same securities in settlement of an unrelated purchase. Such “daisy chain” fails can be cured by borrowing the securities that failed to arrive and delivering the borrowed securities. The dealer returns the borrowed securities when it receives the securities that it purchased.

Concern over settlement fails led the Federal Reserve to initiate a program to lend Treasury securities from its System Open Market Account in 1969. The Fed revised the program in 1999 by, among other things, introducing daily auctions to make loan pricing a competitive process. It holds auctions at noon each day, after the period of greatest liquidity in the over-the-counter securities lending market, to give dealers access to a supplementary source of securities relatively late in the day. The program has become an important part of the Treasury market with securities lending by the Fed averaging $1.5 billion per day between April 1999 and November 2002.

This paper examines the relationship between the Fed’s securities lending program and the private securities lending market. We test whether, and the extent to which, dealers borrow securities from the Fed when prices in the private market suggest they should, and whether they do not borrow securities from the Fed when private sector prices suggest they should not. We also test whether fees paid by dealers in the Fed's auctions are consistent with prices observed in the private market.

Our results show that the markets are reasonably well integrated. Dealers tend to borrow securities from the Fed when the cost of borrowing securities in the private market exceeds the Fed's minimum loan fee, and they tend not to borrow securities from the Fed when the private market cost is less than the Fed's minimum fee. The fees dealers pay the Fed to borrow securities are fairly close to the costs of borrowing the securities in the private market. However, we also present evidence that transaction costs, the design of the Fed’s program, and other market frictions sometimes lead dealers to borrow securities from the Fed even when private sector loan costs are lower, and that they sometimes deter dealers from exploiting arbitrage opportunities by borrowing from the Fed and lending in the private market.
The paper proceeds as follows. Section 1 reviews the mechanisms through which market participants borrow Treasury securities. Section 2 describes the history and current structure of the Fed’s securities lending program. Section 3 explains our data. Section 4 presents our hypotheses, models, and findings regarding dealer behavior in the Fed's securities loan auctions. Section 5 concludes.

I. Borrowing Treasury Securities

Most market participants lend and borrow Treasury securities through repurchase agreements ("repos" or "RPs") and reverse repurchase agreements. A participant executing an RP sells securities (typically for same-day settlement) and simultaneously agrees to repurchase the same securities from the buyer at a higher price on a future date. The transaction is tantamount to borrowing money against a loan of securities, where the proceeds of the sale is the principal amount of the borrowing and the excess of the repurchase price over the sale price is the interest paid on the borrowing. The counterparty to the transaction executes a reverse RP, borrowing (or “reversing in”) securities against lending money.

There are two types of RPs. A *general collateral RP* is an RP in which the lender of funds is willing to accept any of a variety of Treasury securities as collateral. The lender is concerned primarily with earning interest on its money and having possession of securities that can be sold quickly in the event of a default by the borrower. Interest rates on overnight general collateral RPs are usually quite close to contemporaneous rates on overnight loans in the federal (fed) funds market. This reflects the essential character of a general collateral RP as a device for borrowing and lending money.

A *special collateral RP* is an RP in which the lender of funds wants to borrow a particular security. It is, consequently, a device for borrowing and lending securities rather than borrowing and lending money. The rate on a special collateral RP is commonly called a “specials” rate. The owner of a Treasury security may be induced to lend the security if a
dealer offers the owner an opportunity to borrow money at a specials rate below where it can relend the same funds on a general collateral reverse RP.\footnote{For a more extensive discussion of the specials market, see Duffie (1996), Keane (1996), Jordan and Jordan (1997), Buraschi and Menini (2002), and Fisher (2002).}

The difference between the general collateral rate and the specials rate for a security is a measure of the “specialness” of the security. If the demand to borrow the security is modest relative to the supply available for lending, a borrower of the security will usually be able to lend its money at a rate no lower than about 1/8 to 1/4% below the general collateral rate. If the demand to borrow is strong, or if the supply of the security available for lending is limited, the specials rate for the security may be materially below the general collateral rate and the specialness spread correspondingly large; the security is then said to be “on special.” Cornell and Shapiro (1989) and others document instances of large specialness spreads.

Market participants also borrow Treasury securities by pledging collateral and paying a fee to a lender. This arrangement accommodates investors who cannot borrow money (as is the case with some institutional investors) and thus cannot lend securities on an RP. Pledging collateral and paying a fee to a securities lender is economically equivalent to reversing in the securities on a special collateral reverse RP and funding the money lent by borrowing money on a general collateral RP. Consequently, the fee paid to borrow a security is typically about the same as the contemporaneous specialness spread for the security.

The repo and securities lending markets are over-the-counter markets. Some repo market participants run “matched books,” acting as market-makers by quoting bid and offer rates and providing liquidity to others (Bank for International Settlements, 1999, pp. 5-7). As in the market for outright purchases and sales of Treasury securities, brokers arrange transactions between dealers on a blind, or undisclosed, basis. Most securities loans and RPs are for same-day settlement. Trading begins at about 7:00 a.m. New York time and the markets
remain quite active until about 10:30 a.m. Liquidity declines after 10:30 a.m. as participants begin to settle the day’s commitments by delivering and receiving securities.

II. The Federal Reserve’s Securities Lending Program

The idea of a Federal Reserve securities lending program can be traced back to the 1950’s. Ownership of intermediate- and long-term Treasury securities was migrating from banks to pension funds and other emerging classes of institutional investors and some of the new money managers were slow in obtaining authorization to lend the investments that they managed. The shrinking stock of loanable securities led to a rising incidence of settlement fails. The fails problem in turn led to a decline in market liquidity as dealers became reluctant to accommodate customer purchase interests if they did not already have possession of the securities that customers wanted to buy.\(^6\) Some market participants advanced proposals to allow dealers to borrow securities from the single largest owner of Treasury securities, the Federal Reserve’s System Open Market Account (SOMA),\(^7\) but none came to fruition until 1969.

A. The 1969 Program

The spring of 1969 witnessed a sharp increase in fails. To deal with the problem, dealers began to stretch out delivery times and declined to enter into transactions for same-day settlement with other than their best customers. The SOMA manager expressed concern that further degradation of the settlement process might make dealers reluctant to trade with the Fed for same-day settlement and thus might impair the ability of the Fed to achieve its policy


objectives. These concerns provided the impetus for the Federal Open Market Committee (FOMC) to approve a modest securities lending program.\(^8\)

The 1969 program provided for lending Treasury securities on a demand, or “tap,” basis to dealers with a trading relationship with the Fed – so-called “primary” dealers – at a fixed fee of 75 basis points per annum for a term of up to three business days. The program was not intended to be an instrument of monetary policy and was designed to function without affecting the market for reserve balances. In particular, the Fed did not lend securities against borrowing money on RPs (which would have had the effect of draining reserves from the banking system). Instead, dealers had to collateralize their borrowings with other Treasury securities of comparable value.

The 1969 program limited dealer borrowings in several respects. A dealer could not borrow more than $50 million face amount of any single bill or more than $10 million principal amount of any note or bond, and could not borrow more than $75 million of securities in aggregate.\(^9\) Additionally, a dealer had to certify that it was borrowing to replace securities that a seller had failed to deliver and not to finance a short sale, and that it had not been able to borrow the securities elsewhere. The certifications were intended to limit loans to a purpose – avoiding and curing settlement fails – closely associated with the execution of monetary policy and to limit the Fed’s role to that of lender of last resort. The loan fee was set 25 basis points higher than the 50 basis point fee usually charged by private lenders to further limit competition with those lenders and to reinforce the Fed’s position as a lender of last resort.


\(^9\) The lending limits were suspended briefly when Drysdale Government Securities, Inc. failed (Wall Street Journal, 1982), when Hurricane Gloria disrupted New York financial markets in 1985 (Federal Open Market Committee, 1985), and following the break in the stock market in 1987 (Federal Open Market Committee, 1987).
B. The 1999 Revision

As a consequence of the growth in trading of Treasury securities and in the size and complexity of the securities lending market during the 1980s and 1990s, the Federal Reserve revised its lending program in April 1999. Dealer limits were increased substantially and an auction mechanism was introduced to make loan pricing and security allocation a competitive process. The certification requirements were eliminated, but loans were limited to a single business day to make the program less attractive to dealers financing short sales for a purpose other than to accommodate customer purchase interests.  

At noon each business day the Fed offers to lend up to 65% of the amount of each Treasury security beneficially owned by SOMA, subject to an upper limit of the amount of the issue actually in SOMA’s account, i.e., not already out on loan. (As shown in Table 1, the terms of the program have been modified several times since 1999. The text describes program provisions as of the end of our sample period in November 2002.) Primary dealers bid for a loan of a specific security by specifying the quantity desired (in increments of $1 million) and a loan fee. The Fed imposes a minimum fee of 1% to deter dealers from borrowing securities from the Fed that are not on special and that are readily available from private lenders. Bids are accepted until 12:15 p.m.

After the close of an auction, loans are awarded to the highest bidders at their bid rates until all of the securities available for lending have been allocated or all of the bidders have been satisfied (so the auction is a multiple-price, rather than single-price, auction). Awards are subject to the limitations that a dealer cannot borrow more than $200 million of any single issue or more than $1 billion of securities in aggregate. Within minutes of the auction close the Fed informs each participating dealer which of its bids were accepted and which were rejected.

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10 Federal Reserve Bank of New York (1999). Short sales to accommodate customer purchase interests are usually covered within a day or two and are commonly financed with overnight borrowings. Short sales executed as a hedge or as part of a spread trade are commonly financed with open or term borrowings.
It also announces publicly the total amount lent of each security and the weighted average loan fee for each security. Table 2 gives an example of the auction results made public.

C. Why Dealers Borrow Securities from the Federal Reserve

Dealers borrow securities from the Fed to satisfy late-appearing borrowing demands, to reduce their financing costs, and to earn arbitrage profits. As noted above, the over-the-counter markets in securities loans and special collateral RPs are less active at midday compared to 8:00 or 9:00 a.m. If a dealer determines late in the morning that it needs to borrow a security, it may not be able to locate an owner willing and able to deliver the required quantity the same day. The dealer may then choose to submit a bid in the Fed’s auction. The Fed purposely set the time of its auctions after the interval of greatest activity in the private collateral markets to give dealers relatively late access to a supplementary source of securities.

Additionally, dealers anticipate auction outcomes when they borrow and lend collateral earlier in the day. In particular, they seek to lend securities at specials rates lower than those consistent with expected auction loan fees (thereby attempting to earn arbitrage profits) and they abstain from borrowing needed securities at such rates (thereby attempting to reduce their financing costs). A dealer might, for example, lend a security (or abstain from borrowing a security) at a specials rate of 1% when the general collateral rate is 3% (i.e., at a specialness spread of 2%) if it thinks it will be able to borrow the same security from the Fed for a fee of 1-1/4%.

III. Data

Our primary data set consists of the publicly announced results of the Federal Reserve's securities loan auctions from April 26, 1999 to November 6, 2002, including the aggregate
amount of, and average loan fee for, each security lent.\footnote{This information is posted after each auction to the New York Fed's website at www.newyorkfed.org/pihome/statistics/domestic.shtml. Historical data are available at ftp.ny.frb.org/seclend/.} (Fee and quantity data on loans to individual dealers are not publicly available, nor are data on individual or aggregate dealer bids.) The sample interval begins with the first auction under the revised lending program and ends the day the FOMC lowered the target fed funds rate to 1-1/4%. The latter action left only a 1/4\% gap between the general collateral rate – the maximum fee that market participants are generally willing to pay to borrow a security – and the minimum loan fee of 1%.

On average, the Fed lent $1.5 billion principal amount of securities, and 4.6 different issues, per day over the sample period. There was no lending on 38 (4.3\%) of the 892 days on which auctions were conducted. Lending peaked in terms of number of issues (69) and amount lent ($13.4 billion) on September 11 and September 27, 2001, respectively.\footnote{Fleming and Garbade (2002) explore the settlement problems and related surge in securities lending that followed the attacks of September 11.} The average loan fee equaled the minimum loan fee on 2,354 (57.3\%) of the 4,105 security-days in the sample and it was within 10 basis points of the minimum loan fee on an additional 833 (20.3\%) security-days. The Fed lent securities at fees between 10 and 50 basis points over the minimum fee on 397 (9.7\%) occasions, at between 50 and 100 basis points on 166 (4.0\%) occasions, and at more than 100 basis points over the minimum fee on 355 (8.6\%) occasions.

Securities lending activity is highly concentrated in the on-the-run and first off-the-run notes, reflecting the frequency with which these securities trade on special compared to other securities.\footnote{On-the-run securities are the most recently issued securities of a given maturity. First off-the-run securities are the second most recently issued securities of a given maturity.} The on-the-run 2-, 5-, and 10-year notes account for 56.1\% of the quantity of securities lent over the sample period, even though the three issues represent only a small fraction of the approximately 200 issues outstanding on any given day.\footnote{Not including inflation-indexed securities, there were 229 Treasury bills, notes, and bonds outstanding on April 26, 1999 and 167 issues outstanding on November 6, 2002.} First off-the-run notes
account for an additional 16.4% of lending and other notes for 14.0%. Bonds account for only 2.4% of lending, reflecting the fact that bonds rarely traded on special over the sample period. Bills account for the remaining 11.1% of lending; bills do not typically trade on special (Fleming, 2002, p. 711).

Our second data set contains the general collateral repo rate and the special collateral repo rates for the on-the-run and first off-the-run 2-, 5-, and 10-year notes as of 10:00 a.m. each day. The rates are bid rates based on surveys of randomly selected dealers and reported by GovPX through online information vendors. This data is available for 802 days from July 15, 1999 to November 6, 2002, excluding the interval from September 11 to October 9, 2001.

IV. The Relationship between the Federal Reserve’s Securities Loan Auctions and the Over-the-Counter Market for Special Collateral RPs

We use the aforementioned data to test three hypotheses regarding the relationship between the Federal Reserve's securities loan auctions and the over-the-counter market for special collateral RPs. First, we test whether the incidence of borrowing from the Fed is consistent with dealers participating in an auction when, and only when, participation affords them an opportunity to lower their financing costs or earn riskless arbitrage profits. Second, we test whether the quantity of borrowings from the Fed is consistent with dealers fully exploiting opportunities for cost savings and arbitrage profits. Third, we test whether fees paid by dealers in the Fed's securities loan auctions are close to the costs of borrowing the same securities in the over-the-counter market.

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15 The bill maturing March 1, 2001 accounts for 35.1% of all bill lending over the sample period. This bill was the last 52-week bill issued on a 4-week cycle before Treasury reduced the issuance frequency to every 13 weeks. The bill became quite expensive in the special collateral repo market (Fleming, 2000).

16 We also have repo rate data as of 8:10 a.m., 8:30 a.m., and 9:00 a.m. each day, but use the 10:00 a.m. data as it is closest in time to the Fed's securities loan auctions. Surveys are not conducted after 10:00 a.m. because the over-the-counter repo markets are less liquid, although still open, after 10:00 a.m. Results using the earlier data are qualitatively similar and available from the authors.
A. Loan Incidence

We conjecture that the proximate determinant of whether a dealer borrows a security from the Federal Reserve is the cost of borrowing the security in the over-the-counter collateral market, i.e., the specialness spread between the general collateral repo rate and the security’s special collateral repo rate. In particular, we hypothesize that at least one dealer will borrow a security from the Fed if the specialness spread exceeds the Fed's minimum loan fee and that no dealer will borrow a security from the Fed if the specialness spread is less than the minimum loan fee.

We can express our model of loan incidence in an analytical form. Let $\Phi$ be an indicator variable such that $\Phi = 1$ if at least one dealer borrows a particular security from the Fed on a given day and $\Phi = 0$ if no dealer borrows the security. Let $\tilde{S}$ denote the specialness spread for the security at the time of the securities loan auction and let $F_{\text{min}}$ denote the Fed’s minimum loan fee. Ignoring transaction costs and other market frictions, $\Phi$ should depend on $\tilde{S}$ and $F_{\text{min}}$ as:

$$
\Phi = \begin{cases} 
0 & \text{if } \tilde{S} < F_{\text{min}} \\
1 & \text{if } \tilde{S} > F_{\text{min}} 
\end{cases}
$$

(1)

If $\tilde{S} < F_{\text{min}}, \Phi = 0$ because no dealer should be willing to pay more to borrow the security from the Fed than the contemporaneous cost of borrowing the security in the over-the-counter market. Conversely, if $\tilde{S} > F_{\text{min}}$ a dealer who would otherwise pay $\tilde{S}$ to borrow the security in the over-the-counter market may be able to reduce its financing costs by submitting an auction bid to borrow the issue at $F_{\text{min}},$ so $\Phi = 1$. Additionally, if $\tilde{S} > F_{\text{min}}$ a dealer with no ex ante reason to borrow the security but who nevertheless submits an auction bid at $F_{\text{min}}$ may be able to make a riskless profit. If competitors trump its bid and exhaust the amount of the security available for lending, nothing is lost beyond the (trivial) effort it took to submit the bid. However, if the bid at $F_{\text{min}}$ is even partially successful the dealer will be able to relend the
borrowed security on a special collateral RP (and then relend the funds borrowed on the special collateral RP on a general collateral reverse RP) at a specialness spread in excess of the fee paid to the Fed.

We cannot test the model in equation (1) directly because the specialness spreads in our data set are observed at 10:00 a.m. rather than the (noon) time of the Fed’s securities loan auctions. However, let $S$ denote the spread at 10:00 a.m. and suppose, for simplicity, that $\tilde{S} = S + \xi$, where $\xi$ is a normally distributed random variable with mean zero and variance $\omega^2$. \textsuperscript{17} We can then write equation (1) as:

$$
\Phi = \begin{cases} 
0 & \text{if } S + \xi < F_{\min} \\
1 & \text{if } S + \xi > F_{\min}
\end{cases} \quad (2)
$$

It follows that, conditional on the difference between $S$ and $F_{\min}$, the probability that $\Phi = 1$ is:

$$
\Pr[\Phi = 1 \mid S - F_{\min}] = \int_{-\infty}^{-\frac{(S - F_{\min})}{\omega}} n(z) \cdot dz \quad (3)
$$

where $n(\bullet)$ is a standard normal probability density function.

Equation (3) is a special case of a probit model (Greene, 1997, ch. 19):

$$
\Pr[\Phi = 1 \mid S - F_{\min}] = \int_{-\infty}^{-\frac{(S - F_{\min})}{\omega}} n(z) \cdot dz \quad (4)
$$

\textsuperscript{17} Our assumption that $\xi$ is normally distributed is convenient for exposition but cannot be literally true. A specialness spread cannot go below zero and will hardly ever exceed the general collateral rate (because the security’s special collateral repo rate will not – outside of highly unusual circumstances – go below zero and cannot exceed the general collateral rate). It follows that $\xi$ cannot be less than $-S$ (if $\xi < -S$ then $\tilde{S} < 0$) nor greater than the security’s specials rate (if $\xi > Rs$, then $\tilde{S} > S + Rs = Rg$, where $Rg$ is the general collateral rate and $Rs$ is the specials rate).
Comparing equations (3) and (4) shows that we expect $\beta_0 = 0$ and that we can identify $\beta_1$ as $\omega^{-1}$. The 10:00 a.m. specialness spread marking the boundary between when borrowing from the Fed is less likely than not and when borrowing from the Fed is more likely than not, i.e., the specialness spread at which the probability of borrowing is just 50%, is $S_{50\%} = F_{\text{min}} - \beta_0/\beta_1$. $S_{50\%}$ is equal to the minimum loan fee if $\beta_0$ has the expected value of zero.

Table 3 presents estimates of the coefficients of equation (4). The estimated value of $\beta_1$ is positive and highly significant, showing that, as expected, the incidence of dealer borrowing increases with a security's specialness spread. Figure 1 plots the fitted probability of borrowing from the Fed and the actual frequency of borrowing against the difference between the 10:00 a.m. specialness spread and the minimum loan fee. The actual borrowing frequencies track the fitted probabilities fairly well. This evidence suggests that the Fed's securities loan auctions are reasonably well integrated with the over-the-counter market for special collateral RPs.

Contrary to expectations, however, the estimated value of $\beta_0$ in Table 3 is significantly greater than zero. The transition from a market in which borrowing from the Fed is less likely than not to one in which it is more likely than not occurs when the 10:00 a.m. specialness spread is about 25 basis points below the minimum loan fee ($S_{50\%} = F_{\text{min}} - 0.25$ when $\beta_0 = 0.5014$ and $\beta_1 = 2.0246$). The lower than expected transition point is also evident in Figure 1. This finding may reflect a willingness of dealers to pay a (modest) premium for certainty of delivery when securities are needed to deliver against short futures contracts or guaranteed delivery sales to customers. (Unlike private market lenders, the Fed never fails to deliver collateral.)

The positive and statistically significant value of $\beta_0$ may also be attributable to late-appearing borrowing demands. Suppose a dealer determines late in the morning that it needs to borrow a security in a quantity that cannot be satisfied readily (due to market illiquidity) at a price near the specialness spread prevailing in the over-the-counter collateral market. It may then decide to bid for its entire requirement in the Fed's auction. The late-appearing demand does not affect prices in the over-the-counter market when it appears (because it it never
exposed to that market), so the prospect of such a demand cannot affect market prices at 10:00 a.m. Thus, from a statistical point of view, dealers appear surprisingly willing to borrow securities from the Fed when the 10:00 a.m. specialness spread is somewhat below the Fed’s minimum loan fee.

B. Quantity Borrowed

We conjecture that a security’s specialness spread is also the proximate determinant of the quantity of the security borrowed from the Federal Reserve. In particular, we hypothesize that, in the absence of transaction costs and other market frictions, dealers will borrow as much as they can from the Fed if the specialness spread at the time of a securities loan auction exceeds the Fed’s minimum loan fee. However, we also recognize that transaction costs may limit the ability of a dealer to profit from small differences between specialness spreads and the minimum loan fee and hence limit demand for borrowing from the Fed in such instances.

To express our hypothesis in an analytical form, let $Q$ denote the total amount of a security borrowed from the Fed on a given day and let $Q_{\text{max}}$ denote the maximum amount of the security that could have been borrowed. $Q_{\text{max}}$ is the lesser of (1) the maximum amount of the security that the Fed can lend (the product of the maximum fraction of a SOMA position that can be lent and the size of the SOMA position) and (2) the maximum amount of the security that dealers in aggregate can borrow (the product of the maximum amount of the security that a single dealer can borrow and the number of primary dealers).\textsuperscript{18} Ignoring transaction costs and other market frictions, the amount of the security borrowed should depend on the contemporaneous specialness spread for the security as:

\textsuperscript{18} As mentioned, dealers are also limited in the quantity of securities they can borrow in aggregate, but we do not have data on individual dealer borrowings and thus cannot model this constraint.
\[ Q = \begin{cases} 
0 & \text{if } \tilde{S} < F_{\text{min}} \\
Q_{\text{max}} & \text{if } \tilde{S} > F_{\text{min}} 
\end{cases} \quad (5) \]

No dealer will borrow the security from the Fed if it is cheaper to borrow in the over-the-counter market, so \( Q = 0 \) if \( \tilde{S} < F_{\text{min}} \). On the other hand, if \( \tilde{S} > F_{\text{min}} \) and \( Q < Q_{\text{max}} \), some dealer can bid \( F_{\text{min}} \) for the security, have its bid accepted, and relend the security in the over-the-counter market at a profit, so \( Q = Q_{\text{max}} \) if \( \tilde{S} > F_{\text{min}} \).

Defining the borrowing ratio \( \kappa = Q/Q_{\text{max}} \), we can write equation (5) as:

\[ \kappa = \begin{cases} 
0 & \text{if } \tilde{S} < F_{\text{min}} \\
1 & \text{if } \tilde{S} > F_{\text{min}} 
\end{cases} \quad (6) \]

Comparing this model for the borrowing ratio to the model for \( \Phi \) in equation (1) shows that the borrowing ratio should be a binary function of the specialness spread and perfectly correlated with whether any borrowing occurs, i.e., \( \kappa = 1 \) if and only if \( \Phi = 1 \). In other words, dealers should borrow as much as they can from the Fed if they choose to borrow at all.

**B.1. The Effect of Transaction Costs and Other Market Frictions**

Transaction costs and other market frictions can distort the borrowing function in equation (6) by increasing \( \kappa \) above zero for values of \( \tilde{S} \) somewhat less than \( F_{\text{min}} \) and by depressing \( \kappa \) below one for values of \( \tilde{S} \) somewhat greater than \( F_{\text{min}} \). As noted above, some dealers may be willing to pay \( F_{\text{min}} \) to borrow a security from the Fed even when \( \tilde{S} \) is less than \( F_{\text{min}} \) because they need certainty of delivery or because a late-appearing borrowing demand cannot be satisfied readily at the prevailing specials rate. This implies that \( \kappa \) may be positive even when \( \tilde{S} \) is less than \( F_{\text{min}} \). However, since these events are sporadic and idiosyncratic across dealers, we do not expect that such occurrences would drive \( \kappa \) all the way to one.
When $\tilde{S}$ is marginally above $F_{\text{min}}$, transaction costs may limit auction bids motivated by the prospect of securing "cheap" collateral for relending in the over-the-counter market, thereby depressing $\kappa$ below one. If the difference between $\tilde{S}$ and $F_{\text{min}}$ is 10 basis points, for example, the gross gain on $100$ million of collateral borrowed and lent for one day is only $277.79$ ($277.79$ equals $0.10\%$ of $100$ million divided by $360$ days per year). To secure this gain, a dealer must borrow securities from the Fed, relend the securities on a special collateral RP, and then relend the funds borrowed on the special collateral RP on a general collateral reverse RP. In addition to transaction costs, the dealer incurs the risk that the special collateral RP may fail to settle and that it will subsequently need to secure more costly alternative funding, such as a fed funds borrowing or a bank loan, to finance the general collateral reverse RP.

If different dealers incur different transaction costs and have different risk tolerances, then at values of $\tilde{S}$ somewhat greater than $F_{\text{min}}$ some but not all dealers may borrow securities for relending in the over-the-counter market. Since borrowings by any single dealer are limited, each participating dealer may borrow its maximum amount without pushing aggregate borrowings to $Q_{\text{max}}$ if the subset of borrowing dealers is small or if Fed holdings of an issue are large. Aggregate dealer borrowings as a fraction of $Q_{\text{max}}$ are likely to be an increasing function of $\tilde{S}$ for values of the specialness spread not too much greater than $F_{\text{min}}$.

**B.2. A Borrowings Model**

We estimate the variation of the borrowing ratio with specialness spreads using a two-sided censored (Tobit) regression model (Greene, 1997, ch. 20, Nakamura and Nakamura, 1983, and Rosett and Nelson, 1975). This model is used because while transaction costs and other market frictions are likely to cause $\kappa$ to have values other than zero and one, the borrowing ratio is nevertheless bounded between zero and one. The model is:

$$
\kappa = \begin{cases} 
0 & \text{if } \kappa^* < 0 \\
\kappa^* & \text{if } 0 < \kappa^* < 1 \\
1 & \text{if } \kappa^* > 1 
\end{cases}
$$

(9a)
where:

$$\kappa^* = \gamma_0 + \gamma_1 \cdot (\tilde{S} - F_{\text{min}}) + \tilde{\epsilon}$$

$$\tilde{\epsilon} \sim N(0, \sigma^2_{\tilde{\epsilon}})$$

Recalling that \( \tilde{S} = S + \xi \) (where \( S \) is the 10:00 a.m. specialness spread), we can write the equation for \( \kappa^* \) as:

$$\kappa^* = \gamma_0 + \gamma_1 \cdot (S - F_{\text{min}}) + \epsilon$$

$$\epsilon \sim N(0, \varphi^2)$$ (9b)

where \( \epsilon = \gamma_1 \cdot \xi + \tilde{\epsilon} \) and \( \varphi^2 = \gamma_1^2 \cdot \omega^2 + \sigma^2_{\tilde{\epsilon}} \). We expect that \( \gamma_1 \) is positive, so that a larger specialness spread is associated with greater borrowing from the Fed, and that \( \gamma_0 \) is between zero and one, so that (on average) dealers borrow some (but not all) of the maximum amount possible when the specialness spread is equal to the minimum loan fee. A rough indication of the importance of transaction costs and other market frictions can be derived by noting that the expected value of the uncensored borrowing ratio rises linearly from zero when the specialness spread is equal to \( S_0 = F_{\text{min}} - \gamma_0/\gamma_1 \) to one when the specialness spread is equal to \( S_1 = F_{\text{min}} + (1 - \gamma_0)/\gamma_1 \).

Table 4 presents estimates of \( \gamma_0 \) and \( \gamma_1 \). The estimated value of \( \gamma_1 \) is positive and highly significant, showing that, as expected, the borrowing ratio increases with the specialness spread. Also as expected, \( \gamma_0 \) is between zero and one and significantly different from both. The expected value of the uncensored borrowing ratio is zero at a specialness spread 11 basis points less than the minimum fee (\( S_0 = F_{\text{min}} - 0.11 \) when \( \gamma_0 = 0.1049 \) and \( \gamma_1 = 0.9870 \)) and it rises to one at a specialness spread 91 basis points greater than the minimum fee (\( S_1 = F_{\text{min}} + 0.91 \)).
Figure 2 plots the expected censored borrowing ratio and the average actual borrowing ratio against the difference between the specialness spread and the minimum loan fee.\textsuperscript{19} Consistent with the findings in Table 4, dealers do not borrow the maximum possible amount of a security over a wide range of specialness spreads appreciably above the minimum loan fee. Figure 2 also shows that the average actual borrowing ratio tracks the expected censored borrowing ratio fairly closely.

Evidence on the quantity of securities borrowed reinforces the broad assessment that the Fed's securities loan auctions are reasonably integrated with the over-the-counter market for special collateral RPs as well as the caveat that dealers sometimes borrow securities from the Fed even when specialness spreads in the private market are somewhat less than the minimum loan fee. The evidence also suggests that dealers do not take full advantage of opportunities to borrow securities when specialness spreads exceed the minimum loan fee by less than about

\textsuperscript{19} Following Greene (1997, pp. 960 - 961) and Johnson, Kotz, and Balakrishnan (1994, p. 156), the expected censored borrowing ratio conditional on $S - F_{\min}$ is:

$$E[\kappa | S - F_{\min}] = \left\{ \begin{array}{l}
\gamma_0 + \gamma_1 \cdot (S - F_{\min}) + \\
\frac{n \left( -\gamma_0 - \gamma_1 \cdot (S - F_{\min}) \right)}{\Phi} - \frac{n \left( 1 - \gamma_0 - \gamma_1 \cdot (S - F_{\min}) \right)}{\Phi} \cdot \left\{ \frac{1 - \gamma_0 - \gamma_1 \cdot (S - F_{\min})}{\Phi} \right\}
\end{array} \right\}$$

where $n(\bullet)$ is the standard normal probability density function and $N(\bullet)$ is the standard normal cumulative density function.
1%. The latter result is consistent with the proposition that transaction costs and other market frictions limit arbitrage between the over-the-counter market and the Fed’s auctions.

C. Loan Fee

We conjecture that a security’s specialness spread is also the proximate determinant of the average auction loan fee paid to borrow the security from the Federal Reserve. More specifically, we hypothesize that a security's auction loan fee will equal the security’s specialness spread if the spread exceeds the Fed's minimum loan fee. The auction loan fee will equal the minimum loan fee if the specialness spread is less than the minimum fee.

To express our hypothesis in an analytical form, let \( R \) denote the average auction loan fee for a particular security on a given day. Ignoring market frictions, no dealer should be willing to pay more than the contemporaneous specialness spread and competition among dealers should preclude a dealer from paying less than that spread, so \( R = \tilde{S} \) if \( \tilde{S} > F_{\text{min}} \). \( R \) is undefined if \( \tilde{S} < F_{\text{min}} \) because, in the absence of market frictions, dealers would not borrow a security from the Fed if the specialness spread for the security is less than the minimum loan fee. This can be written as:

\[
R = \begin{cases} 
\text{unobserved} & \text{if } \tilde{S} < F_{\text{min}} \\
\tilde{S} & \text{if } \tilde{S} > F_{\text{min}}
\end{cases}
\]  

As shown earlier, dealers sometimes borrow a security from the Fed when its specialness spread is less than the minimum loan fee. We allow for this possibility by modeling observed auction loan fees as:

\[
R = \begin{cases} 
F_{\text{min}} & \text{if } \tilde{S} < F_{\text{min}} \\
\tilde{S} & \text{if } \tilde{S} > F_{\text{min}}
\end{cases}
\]
This equation is not intended to suggest that a security will necessarily be borrowed if \( \tilde{S} \) is less than \( F_{\min} \), but that the average auction loan fee will be \( F_{\min} \) if a security is borrowed when \( \tilde{S} \) is less than \( F_{\min} \).

Equation (11) is equivalent to a Tobit model for the difference between \( R \) and \( F_{\min} \) with realizations of the endogenous variable less than zero censored at zero:

\[
R - F_{\min} = \begin{cases} 
0 & \text{if } R - F_{\min} < 0 \\
R - F_{\min} & \text{if } R - F_{\min} > 0 
\end{cases} 
\quad (12a)
\]

where:

\[
R - F_{\min} = \delta_0 + \delta_1 \cdot (\tilde{S} - F_{\min}) + \bar{\eta} \quad \bar{\eta} \sim N(0, \sigma^{2}_{\eta})
\]

for the special case when \( \delta_0 = 0, \delta_1 = 1, \) and \( \sigma_{\eta} \) is negligible. Substituting \( S + \xi \) for \( \tilde{S} \) gives:

\[
R - F_{\min} = \hat{\delta}_0 + \hat{\delta}_1 \cdot (S - F_{\min}) + \eta \quad \eta \sim N(0, \phi^{2}_{\eta}) 
\quad (12b)
\]

where \( \eta = \delta_1 \cdot \xi + \bar{\eta} \) and \( \phi^{2}_{\eta} = \delta_1^{2} \cdot \omega^{2} + \sigma^{2}_{\eta} \). Consistent with the proposition that equation (11) is a special case of equations (12a,b), we expect \( \delta_0 = 0 \) and \( \delta_1 = 1 \).

Table 5 shows estimates of \( \delta_0 \) and \( \delta_1 \). The estimated value of \( \delta_1 \) is positive and significantly different from zero but, contrary to expectations, it is significantly less than one. The value of \( \hat{\delta}_1 = 0.83 \) indicates that the expected value of the uncensored loan fee rises about 8 basis points for every 10 basis point increase in the specialness spread. Also contrary to expectations, the estimated value of \( \delta_0 \) is significantly greater than zero. The value of \( \hat{\delta}_0 = 0.05 \) implies that the expected value of the uncensored loan fee is about 5 basis points higher than the minimum loan fee when the specialness spread is equal to the minimum loan fee. Taken together, the estimates of \( \delta_0 \) and \( \delta_1 \) imply that the expected value of the uncensored loan
fee is less than the specialness spread when that spread is more than about 30 basis points above the minimum loan fee.\textsuperscript{20}

Figure 3 plots the expected censored loan fee less the minimum loan fee and the average actual loan fee less the minimum loan fee against the specialness spread less the minimum fee.\textsuperscript{21} Consistent with the findings in Table 5, the average actual loan fee increases with the specialness spread. Also consistent with the findings in Table 5, loan fees tend to be less than specialness spreads when the specialness spreads are more than 30 basis points above the minimum loan fee.

The evidence from auction loan fees also suggests that the Fed's securities loan auctions are fairly well integrated with the over-the-counter market for special collateral RPs. However, auction loan fees tend to be systematically less than specialness spreads at spreads more than 30 basis points above the minimum loan fee. This raises the question as to why dealers do not more fully exploit opportunities for riskless arbitrage profits by lending deeply special collateral in the over-the-counter market and financing their loans by borrowing the same securities from the Fed.

\textsuperscript{20} \frac{E[R * - F_{\text{min}} | S - F_{\text{min}}]}{S - F_{\text{min}}} < \frac{S - F_{\text{min}}}{S - F_{\text{min}}} \text{ when } \delta_0 + \delta_1 \cdot (S - F_{\text{min}}) < \frac{S - F_{\text{min}}}{S - F_{\text{min}}}, \text{ or, equivalently, when } S - F_{\text{min}} > \frac{\delta_0}{\delta_0/(1 - \delta_1)}, \text{ because } E[R * - F_{\text{min}} | S - F_{\text{min}}] = \delta_0 + \delta_1 \cdot (S - F_{\text{min}}). \text{ Substituting the values of } \delta_0 = 0.05 \text{ and } \delta_1 = 0.83 \text{ shows that the expected value of } R^* \text{ is less than } \tilde{S} \text{ when } S - F_{\text{min}} > 0.05/(1 - 0.83) = 0.30.

\textsuperscript{21} Following Greene (1997, pp. 960 - 961), the expected value of the censored difference between the auction loan fee and the minimum loan fee, conditional on \(S - F_{\text{min}}\), is:

\[
E[R - F_{\text{min}} | S - F_{\text{min}}] = \delta_0 + \delta_1 \cdot (S - F_{\text{min}}) + \frac{n \left( -\frac{\delta_0 - \delta_1 \cdot (S - F_{\text{min}})}{\phi} \right)}{1 - N \left( -\frac{\delta_0 - \delta_1 \cdot (S - F_{\text{min}})}{\phi} \right)} \cdot \frac{1 - N \left( -\frac{\delta_0 - \delta_1 \cdot (S - F_{\text{min}})}{\phi} \right)}{1 - N \left( -\frac{\delta_0 - \delta_1 \cdot (S - F_{\text{min}})}{\phi} \right)}
\]
C.1. Programmatic Limitations on Arbitrage

One impediment to dealer arbitrage is the limitation that a dealer cannot borrow more than a specified amount of an issue from the Fed. In an extreme case, when SOMA has a large position in a security, every primary dealer can borrow up to its limit without exhausting what the Fed has available to lend. For example, on March 12, 2002, SOMA owned $6.735 billion of the on-the-run 2-year note, the 3% note of February 29, 2004. The maximum fraction of a SOMA position that could be lent on that date was 45% (see Panel B in Table 1) so the Fed could lend up to $3.301 billion of the note. However, there were only 24 primary dealers at the time and each dealer could borrow no more than $100 million of a given security (see Panel C in Table 1), so primary dealers in aggregate could borrow no more than $2.4 billion of the note. Regardless of the note's specialness spread, every dealer could bid the minimum loan fee and yet be certain that its bid would be accepted in full.\textsuperscript{22}

In less extreme cases a dealer might be able to bid less than the specialness spread and still be reasonably sure that its bid will be accepted. We noted earlier that at levels of the specialness spread just above the minimum loan fee some dealers with high transaction costs might not participate in a Fed auction, even at the minimum fee, allowing other dealers to bid the minimum fee and still be awarded securities. At larger specialness spreads, dispersion of dealer transaction costs might allow some dealers to bid somewhat less than the specialness spread but more than the minimum fee and still be awarded securities.

C.2. A Richer Loan Fee Model

We hypothesize that dealers bid less aggressively when the maximum amount the Fed can lend is relatively large. Let $L_{\text{max}}$ denote the maximum amount of a security that the Fed

\textsuperscript{22} Surprisingly, dealers borrowed $1.816 billion of the note that day at an average auction loan fee of 1.17%, 17 basis points higher than the 1.00% minimum loan fee. There are 45 other instances in our sample where dealers borrowed a security at an average loan fee in excess of the minimum loan fee even though every dealer could have borrowed up to its limit without exhausting what the Fed had available to lend.
can lend (computed as the product of the maximum fraction of a SOMA position that can be lent and the size of the SOMA position, rounded down to the nearest million).  

Let $B_{\text{max}}$ denote the maximum amount that dealers in aggregate can borrow (computed as the product of the maximum amount of a security that a single dealer can borrow and the number of primary dealers).  

Let $H$ denote the lesser of one and the ratio of the maximum amount that the Fed can lend to the maximum amount that dealers can borrow:

$$H = \min[1, \frac{L_{\text{max}}}{B_{\text{max}}}] \quad (13)$$

The variation of the uncensored loan fee with the specialness spread for the security can be modeled as:

$$R^* - F_{\text{min}} = \delta_0 + \delta(H) \cdot (S - F_{\text{min}}) + \eta \quad (14)$$

where $\delta(\bullet)$ is a function that declines monotonically over the interval between zero and one, with $\delta(0) = 1$ and $\delta(1) = 0$.

The model in equation (14) is virtually identical to equation (12b) with $\delta_1 = 1$ when $H$ is near zero and the Fed has relatively few securities to lend. The model collapses to $R^* = F_{\text{min}} + \delta_0 + \eta$ when $H = 1$ and the quantity of securities available for lending is equal to or greater than the maximum amount that dealers can borrow. At intermediate values of $H$, $\delta(H)$ is positive but less than one. In this case the expected value of the uncensored loan fee increases less than one-for-one with the specialness spread. The rate of increase is lower the greater the value of $H$.

---

23 SOMA positions in the notes in our sample range from $0.75$ billion to $7.87$ billion. SOMA positions are posted weekly to the New York Fed's website at www.newyorkfed.org/pihome/statistics/soma.shtml.

24 The number of primary dealers over our sample period ranges from 22 to 30. The dealers are listed on the New York Fed's website at www.newyorkfed.org/pihome/new/opnmkttops.
We examine a piecewise-constant representation of the \( \delta(\bullet) \) function. Let \( D[H; a, b] \) indicate whether the value of \( H \) for a given security is equal to or greater than \( a \) and less than \( b \):

\[
D[H; a, b] = \begin{cases} 
1 & \text{if } a \leq H < b \\
0 & \text{otherwise}
\end{cases}
\] (15)

We can express \( \delta(H) \) as a piecewise-constant function:

\[
\delta(H) = \delta_a \cdot D[H; 0.00, 0.25] + \delta_b \cdot D[H; 0.25, 0.50] + \delta_c \cdot D[H; 0.50, 0.75] \\
+ \delta_d \cdot D[H; 0.75, 1.00] + \delta_e \cdot D[H; 1.00, \infty]
\] (16)

The discussion following equation (14) suggests that \( \delta_a \approx 1, \delta_c \approx 0, \) and that \( \delta_a > \delta_b > \ldots > \delta_c. \)

Substituting the expression for \( \delta(H) \) in equation (16) into equation (14) gives the loan fee model:

\[
R - F_{\min} = \begin{cases} 
0 & \text{if } R - F_{\min} < 0 \\
R - F_{\min} & \text{if } R - F_{\min} > 0
\end{cases}
\] (17a)

where:

\[
R - F_{\min} = \delta_0 + \delta_a \cdot D[H; 0.00, 0.25] \cdot (S - F_{\min}) + \delta_b \cdot D[H; 0.25, 0.50] \cdot (S - F_{\min}) \\
+ \delta_c \cdot D[H; 0.50, 0.75] \cdot (S - F_{\min}) + \delta_d \cdot D[H; 0.75, 1.00] \cdot (S - F_{\min}) \\
+ \delta_e \cdot D[H; 1.00, \infty] \cdot (S - F_{\min}) + \eta
\] (17b)
Table 6 shows estimates of the coefficients of the model of equations (17a,b). The estimated value of \( \delta_a \) is close to (albeit statistically significantly less than) one and the estimated values of \( \delta_b, \delta_c, \) and \( \delta_e \) decline in the expected fashion. (The estimated value of \( \delta_d \) has a large standard error because \( D[H; 0.75, 1.00] = 1 \) for only 38 observations.) The estimated values of \( \delta_c, \delta_d, \) and \( \delta_e \) imply that when the amount of a security that the Fed can lend is more than about 50% of what dealers in aggregate can borrow, i.e., when \( H \) is greater than 0.5, the average auction loan fee for the security tends to be materially less than the specialness spread. If the Fed’s lending limit exceeds the aggregate dealer borrowing limit (so \( H = 1 \)), the average auction loan fee is almost independent of the specialness spread for the security, i.e., \( \delta_e \) is close to (albeit statistically significantly greater than) zero.25 The Fed’s securities loan auctions are thus not well integrated with the over-the-counter securities lending market when SOMA has a large position (relative to aggregate dealer borrowing limits) in a security.

V. Conclusions

This paper examines the relationship between the Federal Reserve’s securities lending program and the over-the-counter market for special collateral repurchase agreements. We find that the markets are reasonably well integrated. Dealers tend to borrow securities from the Fed when the cost of borrowing securities in the private market exceeds the Fed's minimum loan fee, and they tend not to borrow securities from the Fed when the private market cost is less than the Fed's minimum fee. Moreover, the auction loan fees dealers pay the Fed to borrow securities are reasonably close to the costs of borrowing the securities in the private market.

Note that the estimated value of \( \delta_e \) is significantly greater than zero. The observations that go into the estimate of this coefficient have a value of \( H \) equal to one, so that every dealer could have bid the minimum loan fee and still borrowed up to its limit. As noted earlier, there are 46 instances in our sample where dealers borrowed a security at an average loan fee in excess of the minimum loan fee even though every dealer could have borrowed up to its limit without exhausting what the Fed had available to lend.
However, the markets are not linked perfectly. Dealers sometimes borrow issues from the Fed when specialness spreads are less than the Fed's minimum loan fee. This may be because the Fed never fails to deliver securities and it may be attributable to large, late-appearing borrowing needs. Additionally, when specialness spreads are greater than the minimum loan fee, dealers tend to borrow less than the maximum amount that could be borrowed and they tend to pay auction loan fees that are less than specialness spreads. Transaction costs and other market frictions, as well as program limitations on dealer borrowings, seem to deter dealers from fully exploiting apparent inter-market arbitrage opportunities.

Our findings suggest a potentially useful modification to the Fed's lending program. The Fed could relax the dealer borrowing limit in cases where SOMA holdings are large. For example, the limit for a specific security might be amended to equal the greater of (a) $200 million and (b) twice the Fed’s lending limit for that security divided by the number of primary dealers.\textsuperscript{26} The Fed’s lending limit would then never be more than half of the aggregate dealer borrowing limit. This change would allow a higher fraction of SOMA holdings to be lent out when SOMA positions are large, and it would reduce the separation between borrowing rates in the over-the-counter collateral market and in the Fed's auctions.

\textsuperscript{26} If the Fed held $6 billion of an issue in an environment with a 65% lending limit and 25 primary dealers, then each dealer could borrow up to $312 million of the issue ($312 million equals two times 65% of $6 billion divided by 25).
References


Federal Open Market Committee, 1985, Minutes of meeting of October 1, 1985, statement of Peter D. Sternlight.


Table 1 - Changes in the Terms of the Federal Reserve's Securities Lending Program, April 26, 1999 - November 6, 2002

The table reports changes in the terms of the Federal Reserve's securities lending program from its April 26, 1999 revision through November 6, 2002. Program changes are posted at the Federal Reserve Bank of New York's website: www.newyorkfed.org/pihome/news/opnmktops/.

<table>
<thead>
<tr>
<th>Panel A: Minimum Loan Fee</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 26, 1999</td>
</tr>
<tr>
<td>September 18, 2001</td>
</tr>
</tbody>
</table>

| Panel B: Maximum Percent of SOMA Position in a Security that Can be Lent |
|-----------------------------|------------------|
| April 26, 1999              | 25%              |
| September 7, 1999           | 45%              |
| September 27, 2001          | 75%              |
| October 18, 2001            | 45%              |
| May 15, 2002                | 65%              |

<table>
<thead>
<tr>
<th>Panel C: Maximum Amount of a Security that a Dealer Can Borrow</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 26, 1999</td>
</tr>
<tr>
<td>September 11, 2001</td>
</tr>
<tr>
<td>September 18, 2001</td>
</tr>
<tr>
<td>September 27, 2001</td>
</tr>
<tr>
<td>October 3, 2001</td>
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<tr>
<td>October 18, 2001</td>
</tr>
<tr>
<td>May 15, 2002</td>
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<table>
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<tr>
<th>Panel D: Maximum Total Amount of Securities that a Dealer Can Borrow</th>
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</thead>
<tbody>
<tr>
<td>April 26, 1999</td>
</tr>
<tr>
<td>September 11, 2001</td>
</tr>
<tr>
<td>October 18, 2001</td>
</tr>
<tr>
<td>May 15, 2002</td>
</tr>
</tbody>
</table>
Table 2 - Securities Loan Auction Results for February 1, 2000


<table>
<thead>
<tr>
<th>Security Description</th>
<th>Total Amount Lent</th>
<th>Weighted Average Fee</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-1/2% note of March 31, 2000</td>
<td>$17 million</td>
<td>1.50%</td>
</tr>
<tr>
<td>6% note of August 15, 2004</td>
<td>$100 million</td>
<td>1.50%</td>
</tr>
<tr>
<td>5-7/8% note of November 15, 2004</td>
<td>$984 million</td>
<td>2.93%</td>
</tr>
<tr>
<td>6% note of August 15, 2009</td>
<td>$1,540 million</td>
<td>4.17%</td>
</tr>
<tr>
<td>6-1/8% bond of August 15, 2029</td>
<td>$483 million</td>
<td>4.37%</td>
</tr>
</tbody>
</table>
Table 3 - Probit Model Estimates of the Incidence of Borrowing

The table reports estimates of the coefficients of a probit model of the incidence of at least one dealer borrowing a security in a Federal Reserve securities loan auction. The probability that at least one dealer borrows a security ($\Phi = 1$) when the specialness spread for the security at 10:00 a.m. (measured in percent) is $S$ and the Fed’s minimum loan fee (also measured in percent) is $F_{\text{min}}$ is:

$$\Pr[\Phi = 1 \mid S - F_{\text{min}}] = \int_{-\infty}^{\infty} \Phi_0 + \beta_1 (S - F_{\text{min}}) \cdot n(z) \cdot dz$$

There are 4,791 observations. Standard errors of estimates are in parentheses; p-values on the null hypothesis a coefficient is zero are in brackets. $S_{50\%}$, the specialness spread marking the transition from a market in which borrowing from the Fed is less likely than not to one in which it is more likely that not, is the value of $S$ that satisfies the equation $\beta_0 + \beta_1 (S - F_{\text{min}}) = 0.0$, or $S_{50\%} = F_{\text{min}} - \beta_0 / \beta_1$.

<table>
<thead>
<tr>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$S_{50%} - F_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5014</td>
<td>2.0246</td>
<td>$-0.2477$</td>
</tr>
<tr>
<td>(0.0439)</td>
<td>(0.0550)</td>
<td>[&lt;.0001]</td>
</tr>
<tr>
<td>[&lt;.0001]</td>
<td>[&lt;.0001]</td>
<td></td>
</tr>
</tbody>
</table>
Table 4 - Tobit Model Estimates of the Borrowing Ratio

The table reports estimates of the coefficients of a two-sided censored (Tobit) model of the borrowing ratio in a Federal Reserve securities loan auction. The borrowing ratio, denoted $\kappa$, is the ratio of the quantity borrowed (measured in $\text{S millions of par amount}$) to the maximum amount of the security that could be borrowed (also measured in $\text{S millions of par amount}$). The observed borrowing ratio is censored below zero and above one. The exogenous variable is the difference between the 10:00 a.m. specialness spread for the security (measured in percent and denoted $S$) and the Fed’s minimum loan fee (also measured in percent and denoted $F_{\text{min}}$):

$$
\kappa = \begin{cases} 
0 & \text{if } \kappa^* < 0 \\
\kappa^* & \text{if } 0 < \kappa^* < 1 \\
1 & \text{if } \kappa^* > 1 
\end{cases}
$$

$$
\kappa^* = \gamma_0 + \gamma_1 \cdot (S - F_{\text{min}}) + \varepsilon
$$

$\varepsilon \sim N(0, \varphi^2)$

There are 4,791 observations. Standard errors of estimates are in parentheses; p-values on the null hypothesis a coefficient is zero are in brackets. $S_0$, the specialness spread at which the expected value of the uncensored borrowing ratio is zero, is the value of S that satisfies the equation $\gamma_0 + \gamma_1 \cdot (S - F_{\text{min}}) = 0$, or $S_0 = F_{\text{min}} - \gamma_0/\gamma_1$. $S_1$, the specialness spread at which the expected value of the uncensored borrowing ratio is one, is the value of S that satisfies the equation $\gamma_0 + \gamma_1 \cdot (S - F_{\text{min}}) = 1$, or $S_1 = F_{\text{min}} + (1- \gamma_0)/\gamma_1$.

<table>
<thead>
<tr>
<th>$\gamma_0$</th>
<th>$\gamma_1$</th>
<th>$\varphi$</th>
<th>$S_0 - F_{\text{min}}$</th>
<th>$S_1 - F_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1049</td>
<td>0.9870</td>
<td>0.6522</td>
<td>-0.106</td>
<td>0.907</td>
</tr>
<tr>
<td>(0.0187)</td>
<td>(0.0279)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[&lt;.0001]</td>
<td>[&lt;.0001]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

33
Table 5 - Tobit Model Estimates of the Auction Loan Fee

The table reports estimates of the coefficients of a censored (Tobit) model of the difference between the average auction loan fee and the minimum loan fee in a Federal Reserve securities loan auction. The average auction loan fee, denoted $R$, and the minimum loan fee, denoted $F_{min}$, are both measured in percent. The difference is censored below zero. The exogenous variable is the difference between the 10:00 a.m. specialness spread for the security (measured in percent and denoted $S$) and $F_{min}$:

$$R - F_{min} = \begin{cases} 0 & \text{if } R - F_{min} < 0 \\ R - F_{min} & \text{if } R - F_{min} > 0 \end{cases}$$

$$R - F_{min} = \delta_0 + \delta_1 \cdot (S - F_{min}) + \eta \quad \eta \sim N(0, \phi^2)$$

There are 1,176 observations. (The number of observations is smaller than in Tables 3 and 4 because a security must be borrowed to be included in the sample for this table.) Standard errors of estimates are in parentheses; p-values on the null hypotheses $\delta_0$ is zero and $\delta_1$ is one are shown in brackets.

<table>
<thead>
<tr>
<th>$\delta_0$</th>
<th>$\delta_1$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0525</td>
<td>0.8324</td>
<td>0.5871</td>
</tr>
<tr>
<td>(0.0211)</td>
<td>(0.0157)</td>
<td>[&lt;.0001]</td>
</tr>
<tr>
<td>[.0130]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 6 – Extended Tobit Model Estimates of the Auction Loan Fee

The table reports estimates of the coefficients of a censored (Tobit) model of the difference between the average auction loan fee and the minimum loan fee in a Federal Reserve securities loan auction. The average auction loan fee, denoted \( R \), and the minimum loan fee, denoted \( F_{\text{min}} \), are both measured in percent. The difference is censored below zero. The exogenous variable is the difference between the 10:00 a.m. specialness spread for the security (measured in percent and denoted \( S \)) and \( F_{\text{min}} \). The coefficient on the exogenous variable is allowed to vary as a discrete function of the variable \( H \), which is the minimum of one and the ratio of the maximum amount the Fed can lend (\( L_{\text{max}} \), measured in $ millions of par amount) to the maximum amount that dealers in aggregate can borrow (\( B_{\text{max}} \), also measured in $ millions of par amount): \( H = \min[1, L_{\text{max}}/B_{\text{max}}] \). The dummy variable \( D[H; a, b] \) is one if \( a \leq H < b \) and zero otherwise.

\[
R - F_{\text{min}} = \begin{cases} 
0 & \text{if } R - F_{\text{min}} < 0 \\
R - F_{\text{min}} & \text{if } R - F_{\text{min}} > 0
\end{cases}
\]

\[
R - F_{\text{min}} = \delta_0 + \delta_a \cdot D[H; 0.00, 0.25] \cdot (S - F_{\text{min}}) + \delta_b \cdot D[H; 0.25, 0.50] \cdot (S - F_{\text{min}}) + \delta_c \cdot D[H; 0.50, 0.75] \cdot (S - F_{\text{min}}) + \delta_d \cdot D[H; 0.75, 1.00] \cdot (S - F_{\text{min}}) + \delta_e \cdot D[H; 1.00, \infty] \cdot (S - F_{\text{min}}) + \eta
\]

\( \eta \sim N(0, \phi^2) \)

There are 1,176 observations. Standard errors of estimates are in parentheses; p-values on the null hypothesis a coefficient is zero are in brackets.

\[
\begin{array}{cccccccc}
\delta_0 & \delta_a & \delta_b & \delta_c & \delta_d & \delta_e & \phi \\
0.0617 & 0.9251 & 0.8762 & 0.6141 & 0.2514 & 0.2843 & 0.5473 \\
(0.0197) & (0.0228) & (0.0205) & (0.0283) & (0.1821) & (0.1169) & \\
\end{array}
\]
Figure 1 - Fitted Probability and Actual Frequency of Borrowing

The figure plots the fitted probability and actual frequency of at least one dealer borrowing a security in a Federal Reserve securities loan auction against the difference between the specialness spread for the security at 10:00 a.m. (measured in percent) and the Fed's minimum loan fee (also measured in percent). The actual frequency of borrowing is estimated for each 10 basis point interval, inclusive of observations at the lower boundary of the interval.
Figure 2 - Expected Censored and Average Actual Borrowing Ratio

The figure plots the expected censored and average actual borrowing ratio in a Federal Reserve securities loan auction against the difference between the specialness spread for a security at 10:00 a.m. (measured in percent) and the Fed’s minimum loan fee (also measured in percent). The borrowing ratio is the ratio of the quantity borrowed to the maximum amount that could be borrowed. The observed borrowing ratio is censored below zero and above one. The average actual borrowing ratio is estimated for each 10 basis point interval, inclusive of observations at the lower boundary of the interval.
Figure 3 - Expected Censored Auction Loan Fee less the Minimum Loan Fee and Average Actual Auction Loan Fee less the Minimum Loan Fee

The figure plots the expected censored loan auction loan fee less the minimum loan fee and the average actual auction loan fee less the minimum loan fee against the difference between the specialness spread for a security at 10:00 a.m. and the minimum loan fee. All variables are measured in percent. The observed difference between the auction loan fee and the minimum loan fee is censored below zero. The average actual auction loan fee is estimated for each 10 basis point interval, inclusive of observations at the lower boundary of the interval.